

INVERSE TRIGONOMETRIC FUNCTIONS



BASIC CONCEPTS

Principal value branches: Since trigonometric functions being periodic are in general not bijective (one-one onto) and thus for existence of inverse of trigonometric function we restrict their domain and co-domain to make it bijective. This restriction of domain and range gives principal value branch of inverse trigonometric function which are as follows:

Functions	Domain	Range (Principal value branch)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

The value of an inverse trigonometric function which lies in its principal value branch is called the principal value of that inverse trigonometric function.

Properties of Inverse Trigonometric Functions

- $\sin^{-1} x + \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\},$ if $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$
 - $\sin^{-1} x - \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\},$ if $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$
- $\cos^{-1} x + \cos^{-1} y = \cos^{-1} \{xy - \sqrt{1-x^2}\sqrt{1-y^2}\},$ if $-1 \leq x, y \leq 1$ and $x + y \geq 0$
 - $\cos^{-1} x - \cos^{-1} y = \cos^{-1} \{xy + \sqrt{1-x^2}\sqrt{1-y^2}\},$ if $-1 \leq x, y \leq 1$ and $x \leq y$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right),$ if $xy < 1$
 - $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right),$ if $xy > -1$
- $2 \sin^{-1} x = \sin^{-1} (2x\sqrt{1-x^2}),$ if $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
 - $2 \cos^{-1} x = \cos^{-1} (2x^2 - 1),$ if $0 \leq x \leq 1$
 - $2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right),$ if $-1 < x < 1$

5. (i) $3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$, if $-\frac{1}{2} \leq x \leq \frac{1}{2}$
- (ii) $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, if $\frac{1}{2} \leq x \leq 1$
- (iii) $3 \tan^{-1} x = \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right)$, if $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$
6. (i) $2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1 + x^2} \right)$, if $-1 \leq x \leq 1$
- (ii) $2 \tan^{-1} x = \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$, if $0 \leq x < \infty$
7. (i) $\sin^{-1} x = \cos^{-1} (\sqrt{1 - x^2}) = \tan^{-1} \left(\frac{x}{\sqrt{1 - x^2}} \right)$
 $= \cot^{-1} \left(\frac{\sqrt{1 - x^2}}{x} \right) = \sec^{-1} \left(\frac{1}{\sqrt{1 - x^2}} \right) = \operatorname{cosec}^{-1} \left(\frac{1}{x} \right)$
- (ii) $\cos^{-1} x = \sin^{-1} (\sqrt{1 - x^2}) = \tan^{-1} \left(\frac{\sqrt{1 - x^2}}{x} \right)$
 $= \cot^{-1} \left(\frac{x}{\sqrt{1 - x^2}} \right) = \sec^{-1} \left(\frac{1}{x} \right) = \operatorname{cosec}^{-1} \left(\frac{1}{\sqrt{1 - x^2}} \right)$
- (iii) $\tan^{-1} x = \sin^{-1} \left(\frac{x}{\sqrt{1 + x^2}} \right) = \cos^{-1} \left(\frac{1}{\sqrt{1 + x^2}} \right)$
 $= \cot^{-1} \left(\frac{1}{x} \right) = \sec^{-1} (\sqrt{1 + x^2}) = \operatorname{cosec}^{-1} \left(\frac{\sqrt{1 + x^2}}{x} \right)$

Important substitution

Expression	Substitution
$a^2 + x^2$	$x = a \tan \theta$ or $x = a \cot \theta$
$a^2 - x^2$	$x = a \sin \theta$ or $x = a \cos \theta$
$x^2 - a^2$	$x = a \sec \theta$ or $x = a \operatorname{cosec} \theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	$x = a \cos 2\theta$

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. The value of $\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right)$ corresponding to principal branches is

(a) $-\frac{\pi}{12}$ (b) 0 (c) π (d) $\frac{\pi}{3}$

2. The value of $\cot(\sin^{-1} x)$ is

[NCERT Exemplar]

(a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

3. The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is [NCERT Exemplar]
 (a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $\frac{-5\pi}{9}$ (d) $\frac{7\pi}{18}$
4. Let $\theta = \sin^{-1}(\sin(-600^\circ))$, then value of θ is
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$
5. The value of the expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is [NCERT Exemplar]
 (a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1
6. The value of $\tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$ is
 (a) 5 (b) 11 (c) 13 (d) 15
7. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is [NCERT Exemplar]
 (a) $[1, 2]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) none of these
8. The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is [NCERT Exemplar]
 (a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$
9. $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to
 (a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$
10. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals [NCERT Exemplar]
 (a) 0 (b) 1 (c) 6 (d) 12
11. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to
 (a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$
12. The value of $\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}]$ is
 (a) $\sqrt{1-x^2}$ (b) 1 (c) x (d) x^2
13. If $\theta = \sin^{-1}(\sin 600^\circ)$ then the value of θ is
 (a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) 0 (d) $\frac{2\pi}{3}$
14. $\cos^{-1}\left[\cos\frac{7\pi}{6}\right]$ is equal to
 (a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$
15. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1
16. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to
 (a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

17. The domain of $y = \cos^{-1}(x^2 - 4)$ is
 (a) $[-1, 1]$ (b) $[-\sqrt{5}, -\sqrt{3}] \cap [\sqrt{3}, \sqrt{5}]$
 (c) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ (d) $[0, \pi]$
18. The domain of the function defined $f(x) = \sin^{-1} x + \cos x$ is
 (a) ϕ (b) $(-\infty, \infty)$ (c) $[-1, 1]$ (d) $[0, \pi]$
19. The value of $\sin(2 \sin^{-1}(0.8))$ is
 (a) $\sin 1.6$ (b) 1.6 (c) 0.96 (d) 4.8
20. $\sin^{-1}(\sin 5) > x^2 - 4x$ if
 (a) $x = 2 - \sqrt{9 - 2\pi}$ (b) $x = 2 + \sqrt{9 - 2\pi}$
 (c) $x > 2 + \sqrt{9 - 2\pi}$ (d) $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$
21. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ and $f(1) = 1, f(p+q) = f(p) \cdot f(q), \forall p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$ equals
 (a) 2 (b) 1 (c) 0 (d) 3
22. If $u = \cot^{-1} \sqrt{\tan \alpha} - \tan^{-1} \sqrt{\tan \alpha}$, then $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$ is equal to
 (a) $\sqrt{\tan \alpha}$ (b) $\sqrt{\cot \alpha}$ (c) $\tan \alpha$ (d) $\cot \alpha$
23. If $\cos^{-1} x > \sin^{-1} x$, then
 (a) $\frac{1}{\sqrt{2}} < x \leq 1$ (b) $0 \leq x < \frac{1}{\sqrt{2}}$ (c) $-1 \leq x < \frac{1}{\sqrt{2}}$ (d) $x > 0$
24. If $f(x) = \sin^{-1} x$, then domain of $f(x)$ is
 (a) $x \geq 1$ or $x \leq -1$ (b) $-1 \leq x \leq 1$ (c) $x \geq 1$ (d) None of these
25. The domain of the function $y = \sin^{-1}(-x^2)$ is
 (a) $[0, 1]$ (b) $(0, 1)$ (c) $[-1, 1]$ (d) $\emptyset$
26. If $ax + b\{\sec(\tan^{-1} x)\} = c$ and $ay + b\{\sec(\tan^{-1} y)\} = c$, then $\frac{x+y}{1-xy} =$
 (a) $\frac{2ac}{a-c}$ (b) $\frac{ac}{a^2+c^2}$ (c) $\frac{2ac}{a^2-c^2}$ (d) $\frac{a+c}{1-ac}$
27. If $\alpha = \tan^{-1}\left\{\tan\left(\frac{5\pi}{4}\right)\right\}$ and $\beta = \tan^{-1}\left\{-\tan\left(\frac{2\pi}{3}\right)\right\}$ then
 (a) $4\alpha = 3\beta$ (b) $3\alpha = 4\beta$ (c) $\alpha = \beta$ (d) none of these
28. The value of expression $\tan\left(\frac{\sin^{-1} x + \cos^{-1} x}{2}\right)$, where $x = \frac{\sqrt{3}}{2}$ is equal to
 (a) ∞ (b) 1 (c) -1 (d) none of these
29. Domain of $\cos^{-1}[x]$ is
 (a) $[-2, 1]$ (b) $(-1, 1)$ (c) $[-1, 2)$ (d) None of these
30. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ will be invertible for the domain
 (a) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$ (c) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (d) $x \in \left[0, \frac{\pi}{4}\right]$
31. The domain of $\sin^{-1}[x]$ is given by
 (a) $[-1, 1]$ (b) $[-1, 2)$ (c) $\{-1, 0, 1\}$ (d) None of these

32. If $\sin(\pi \cos x) = \cos(\pi \sin x)$ then x equals

- (a) $\frac{1}{2} \sin^{-1} \frac{3}{4}$ (b) $\frac{1}{2} \cos^{-1} \frac{3}{4}$ (c) $-\frac{1}{4} \sin^{-1} \frac{3}{4}$ (d) $-\frac{1}{2} \cos^{-1} \frac{3}{4}$

33. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to

- (a) 4 (b) $2 \sin 2\alpha$ (c) $-4 \sin^2 \alpha$ (d) $4 \sin^2 \alpha$

34. $3 \sin x + 4 \cos x = y^2 - 2y + 6$. If x, y are its solutions then which of the following is true?

- (a) $y = 1$ (b) $x = \frac{\pi}{2} - \tan^{-1} \left(\frac{4}{3} \right)$
(c) $xy = \frac{\pi}{2} - \tan^{-1} \left(\frac{4}{3} \right)$ (d) All of these

35. If $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, then x lies in the interval

- (a) $(\cot 5, \cot 2)$ (b) $(-\infty, \cot 5) \cup (\cot 2, \infty)$
(c) $(-\infty, \cot 5)$ (d) $(\cot 2, \infty)$

36. The domain of $f(x) = \frac{\sin^{-1} x}{x}$ is

- (a) $[-1, 1]$ (b) $\{0\}$ (c) $[-1, 0)$ (d) None of these

37. Let $f(x) = e^{\cos^{-1} \sin(x + \frac{\pi}{3})}$, then $f\left(\frac{8\pi}{9}\right)$ equals

- (a) $e^{\frac{7\pi}{12}}$ (b) $e^{\frac{13\pi}{18}}$ (c) $e^{\frac{5\pi}{18}}$ (d) $e^{\frac{\pi}{12}}$

38. If $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the solution of the equation $\log_{\sin \theta}(\cos^2 \theta - \sin^2 \theta) = 2$ is given by

- (a) $\theta = \sin^{-1} \left(-\frac{1}{\sqrt{3}} \right)$ (b) $\theta = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$ (c) $\theta = n\pi$ (d) $\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$

39. If $\sin^{-1} a + \sin^{-1} b + \sin^{-1} c = \frac{3\pi}{2}$ and $f(2) = 2$, $f(x+y) = f(x)f(y) \forall x, y \in \mathbb{R}$, then

$$a^{f(2)} + b^{f(4)} + c^{f(6)} - 3 \frac{(a^{f(2)} \cdot b^{f(4)} \cdot c^{f(6)})}{a^{f(2)} + b^{f(4)} + c^{f(6)}} \text{ equals}$$

- (a) 2 (b) 4 (c) 6 (d) 8

40. If $\sin^{-1}(x^2 - 7x + 12) = n\pi, \forall n \in \mathbb{Z}$, then x equals

- (a) -4 (b) 4 (c) -3 (d) None of these

41. The value of x satisfies the inequality $[\tan^{-1} x]^2 - 2[\tan^{-1} x] - 3 \leq 0$, where $[.]$ represent greatest integer function, then x lies between

- (a) $[-\tan 1, \infty)$ (b) $[-\tan 1, \tan 3]$ (c) $\left[-\frac{\pi}{4}, \tan^{-1} 3\right]$ (d) $\left[-\frac{\pi}{2}, \tan 1\right]$

42. $\cos^{-1} \left[\cos \left(\left(-\frac{17}{15} \right) \pi \right) \right]$ is equal to

- (a) $\frac{17\pi}{15}$ (b) $\frac{13\pi}{15}$ (c) $\frac{3\pi}{15}$ (d) $-\frac{17\pi}{15}$

43. Which of the following is the principal value branch of $\cos^{-1} x$?

- (a) $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ (b) $(0, \pi)$ (c) $[0, \pi]$ (d) $(0, \pi) - \left\{ \frac{\pi}{2} \right\}$

44. Which of the following is the principal value branch of $\operatorname{cosec}^{-1} x$?

- (a) $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ (b) $(0, \pi) - \left\{ \frac{\pi}{2} \right\}$ (c) $\left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

45. The value of $\sin^{-1} \left[\cos \left(\frac{33\pi}{5} \right) \right]$ is
 (a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$
46. The domain of the function $\cos^{-1} (2x - 1)$ is
 (a) $[0, 1]$ (b) $[-1, 1]$ (c) $(-1, 1)$ (d) $[0, \pi]$
47. The value of $\cos^{-1} \left(\cos \frac{3\pi}{2} \right)$ is
 (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$
48. The value of $2 \sec^{-1} 2 + \sin^{-1} \left(\frac{1}{2} \right)$ is
 (a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1
49. The value of $\cot \left[\cos^{-1} \left(\frac{7}{25} \right) \right]$ is
 (a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$
50. The number of real solutions of the equation $\sqrt{1 + \cos 2x} = \sqrt{2} \cos^{-1} (\cos x)$ in $\left[\frac{\pi}{2}, \pi \right]$ is
 (a) 0 (b) 1 (c) 2 (d) ∞
51. The principal value of $\sin^{-1} \frac{1}{2}$ is
 (a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) Both (a) & (b)
52. The principal value of $\operatorname{cosec}^{-1} (-1)$ is
 (a) $\frac{-\pi}{2}$ (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$
53. The value of $\tan^{-1}(\sqrt{3}) + \cot^{-1}(-1) + \sec^{-1} \left(\frac{-2}{\sqrt{3}} \right)$ is
 (a) $\frac{-\pi}{12}$ (b) $\frac{11\pi}{12}$ (c) $\frac{5\pi}{4}$ (d) $\frac{23\pi}{12}$
54. The value of $2 \cos^{-1} \left(\frac{-1}{2} \right) + 2 \sin^{-1} \left(\frac{-1}{2} \right) - \cos^{-1}(-1)$ is
 (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π
55. The value of $\sec^{-1} \left(\sec \frac{4\pi}{3} \right)$ is
 (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{-\pi}{3}$
56. The value of $\cos^{-1}(-1) + \sin^{-1}(1)$ is
 (a) $\frac{-3\pi}{2}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$
57. The value of $\cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left(\sin \frac{5\pi}{3} \right)$ is equal to
 (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{10\pi}{3}$ (d) $\frac{2\pi}{3}$

58. The value of $\cot\left[\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right]$ is

- (a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0

59. The value of $\sin^{-1}\left[-\left(\frac{1}{2}\right)\right] + \cos^{-1}\left[-\left(\frac{1}{2}\right)\right] + \cot^{-1}(-\sqrt{3}) + \operatorname{cosec}^{-1}(\sqrt{2}) + \tan^{-1}(-1) + \sec^{-1}(\sqrt{2})$ equals

- (a) $\frac{9\pi}{4}$ (b) $\frac{19\pi}{12}$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{2}$

60. The value of $\cos^{-1}(2x^2 - 1)$, $0 \leq x \leq 1$ is equal to

- (a) $2\cos^{-1}x$ (b) $2\sin^{-1}x$ (c) $\pi - 2\cos^{-1}x$ (d) $\pi + 2\cos^{-1}x$

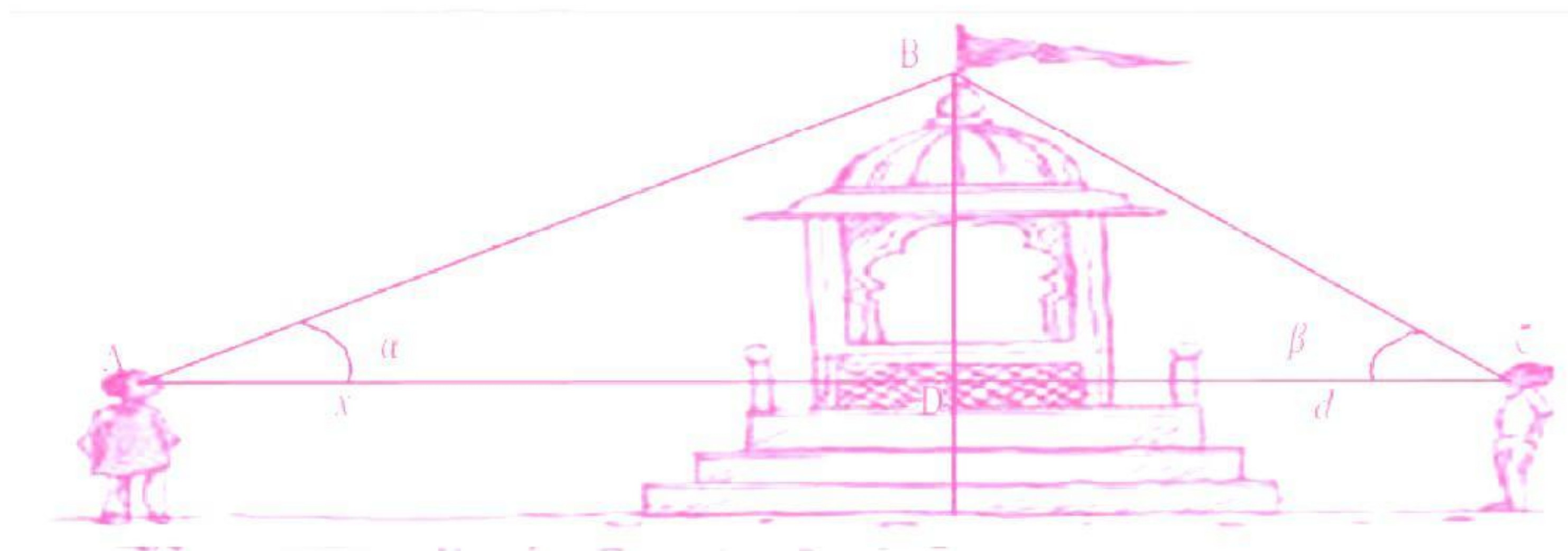
Answers

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (d) | 4. (a) | 5. (b) | 6. (b) |
| 7. (a) | 8. (d) | 9. (d) | 10. (c) | 11. (b) | 12. (c) |
| 13. (b) | 14. (b) | 15. (d) | 16. (b) | 17. (c) | 18. (c) |
| 19. (c) | 20. (d) | 21. (a) | 22. (a) | 23. (c) | 24. (b) |
| 25. (c) | 26. (c) | 27. (a) | 28. (b) | 29. (c) | 30. (b) |
| 31. (b) | 32. (a) | 33. (d) | 34. (d) | 35. (b) | 36. (d) |
| 37. (b) | 38. (b) | 39. (a) | 40. (b) | 41. (a) | 42. (b) |
| 43. (c) | 44. (d) | 45. (d) | 46. (a) | 47. (a) | 48. (b) |
| 49. (d) | 50. (a) | 51. (a) | 52. (a) | 53. (d) | 54. (a) |
| 55. (b) | 56. (d) | 57. (a) | 58. (c) | 59. (b) | 60. (a) |

CASE-BASED QUESTIONS

Choose and write the correct option in the following questions.

1. Read the following and answer any four questions from (i) to (v).



Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following: [CBSE Question Bank]

Answer the questions given below.

(i) $\angle CAB = \alpha =$

- (a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$ (c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$ (c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

(iii) $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}(\sqrt{3})$

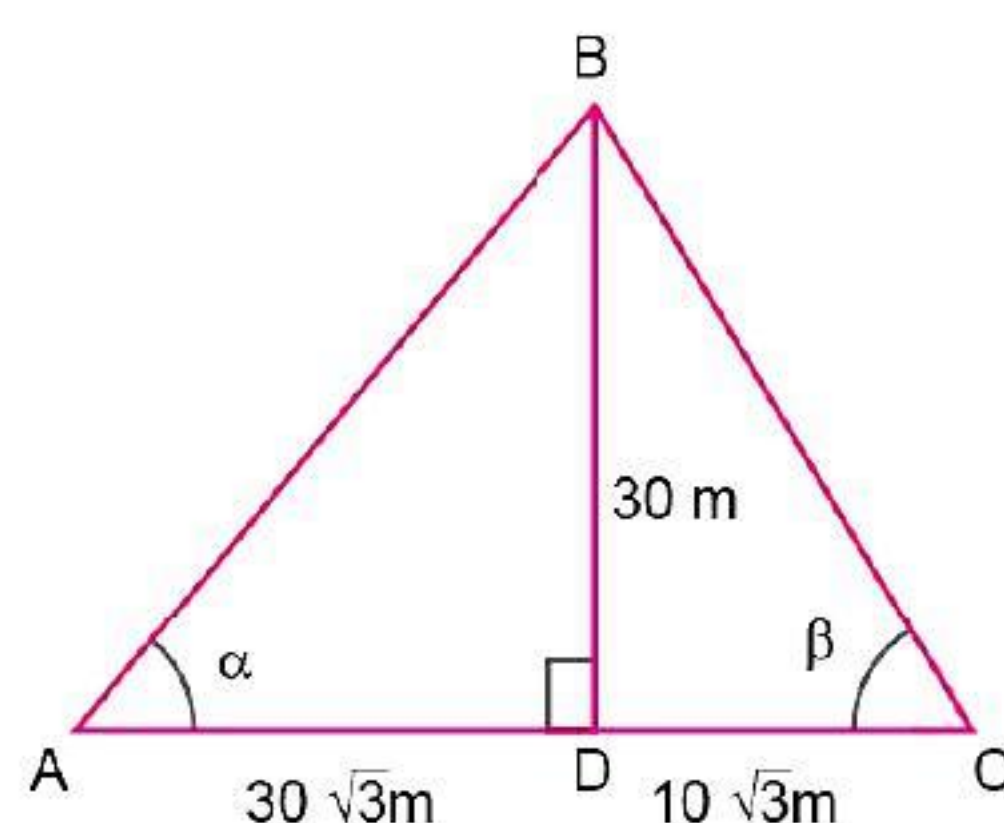
(iv) $\angle ABC =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and Range of $\cos^{-1} x$ are respectively

(a) $(-1, 1), (0, \pi)$ (b) $[-1, 1], (0, \pi)$ (c) $[-1, 1], [0, \pi]$ (d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Sol. We have,



(i) Now in $\triangle ABD$ (right angled)

$$\tan \alpha = \frac{BD}{AD} = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \tan \alpha = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \alpha = 30^\circ$$

$$\Rightarrow \sin \alpha = \sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$\therefore$ Option (b) is correct.

(ii) We have from (i) $\alpha = 30^\circ \Rightarrow \cos \alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ Option (c) is correct.

(iii) In right $\triangle BCD$, we have

$$\tan \beta = \frac{BD}{DC} \Rightarrow \tan \beta = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \beta = \tan^{-1}(\sqrt{3})$$

$\therefore$ Option (d) is correct.

(iv) In $\triangle ABC$, we have,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ABC + \alpha + \beta = 180^\circ \Rightarrow \angle ABC + 30^\circ + 60^\circ = 180^\circ \text{ [From (iii) } \beta = \tan^{-1}(\sqrt{3}) = 60^\circ \text{]}$$

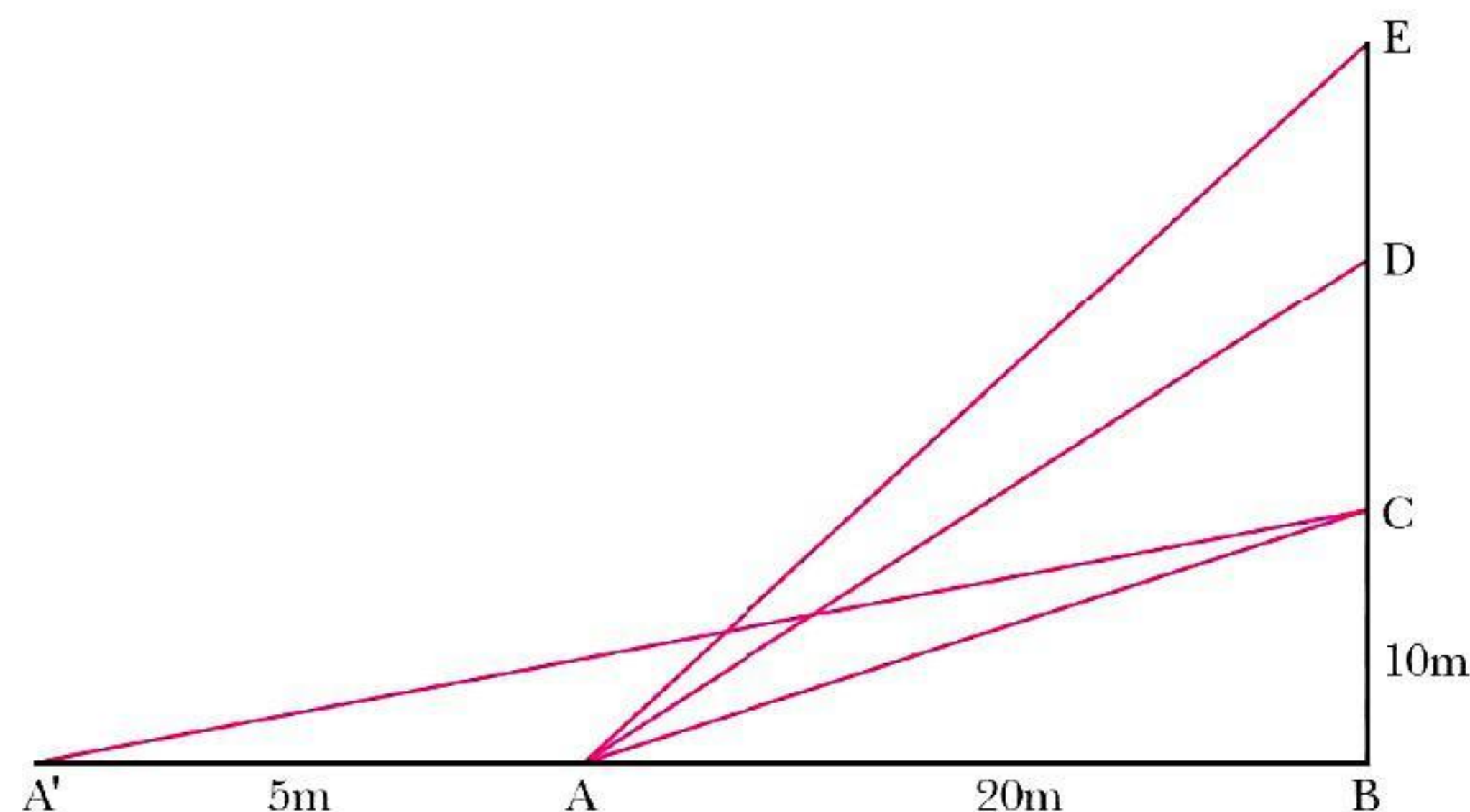
$$\Rightarrow \angle ABC = 90^\circ$$

$$\Rightarrow \angle ABC = \frac{\pi}{2}$$

$\therefore$ Option (c) is correct.

- (v) Let $\cos^{-1} x = y \Rightarrow x = \cos y$
 $-1 \leq \cos y \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow \text{Domain} = [-1, 1]$
 $0 \leq y \leq \pi \Rightarrow \text{Range} = [0, \pi].$
 $\therefore$ Option (c) is correct.

2. Read the following and answer any four questions from (i) to (v).



The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C". The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following: [CBSE Question Bank]

Answer the questions given below.

(i) Measure of $\angle CAB =$

- (a) $\tan^{-1}(2)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$ (c) $\tan^{-1}(1)$ (d) $\tan^{-1}(3)$

(ii) Measure of $\angle DAB =$

- (a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}(3)$ (c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}(4)$

(iii) Measure of $\angle EAB =$

- (a) $\tan^{-1}(11)$ (b) $\tan^{-1}3$ (c) $\tan^{-1}\left(\frac{2}{11}\right)$ (d) $\tan^{-1}\left(\frac{11}{2}\right)$

(iv) A' Is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$ is

- (a) $\tan^{-1}\left(\frac{1}{12}\right)$ (b) $\tan^{-1}\left(\frac{1}{8}\right)$ (c) $\tan^{-1}\left(\frac{2}{5}\right)$ (d) $\tan^{-1}\left(\frac{11}{21}\right)$

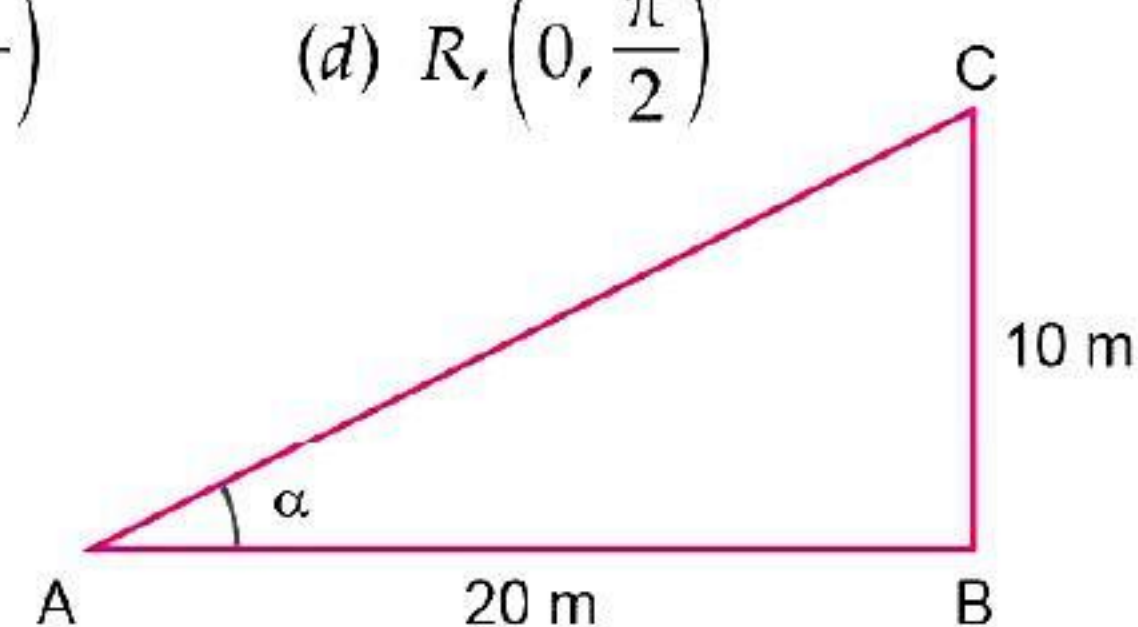
(v) Domain and Range of $\tan^{-1} x$ are respectively

- (a) $R^+, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $R^-, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $R, \left(0, \frac{\pi}{2}\right)$

Sol. Let $\angle CAB = \alpha$, therefor $\angle DAB = 2\alpha$ and $\angle EAB = 3\alpha$.

(i) In right $\triangle ABC$, we have

$$\tan \alpha = \frac{BC}{AB} = \frac{10}{20} = \frac{1}{2}$$



$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \angle CAB = \tan^{-1}\left(\frac{1}{2}\right)$$

$\therefore$ Option (b) is correct.

(ii) We have, $\tan \alpha = \frac{1}{2}$ (from (i))

$$\therefore \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}$$

$$\tan 2\alpha = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow 2\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\Rightarrow \angle DAB = \tan^{-1}\left(\frac{4}{3}\right)$$

$\therefore$ Option (c) is correct.

(iii) We have, $\tan \alpha = \frac{1}{2}$

$$\therefore \tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$= \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3 \times \left(\frac{1}{2}\right)^2}$$

$$\tan 3\alpha = \frac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{8} \times 4 = \frac{11}{2}$$

$$\Rightarrow 3\alpha = \tan^{-1}\left(\frac{11}{2}\right) \Rightarrow \angle EAB = \tan^{-1}\left(\frac{11}{2}\right).$$

$\therefore$ Option (d) is correct.

(iv) Let $\angle CA'B = \beta$

We have, In $A'CB$

$$\tan \beta = \frac{BC}{A'B} = \frac{10}{20+5}$$

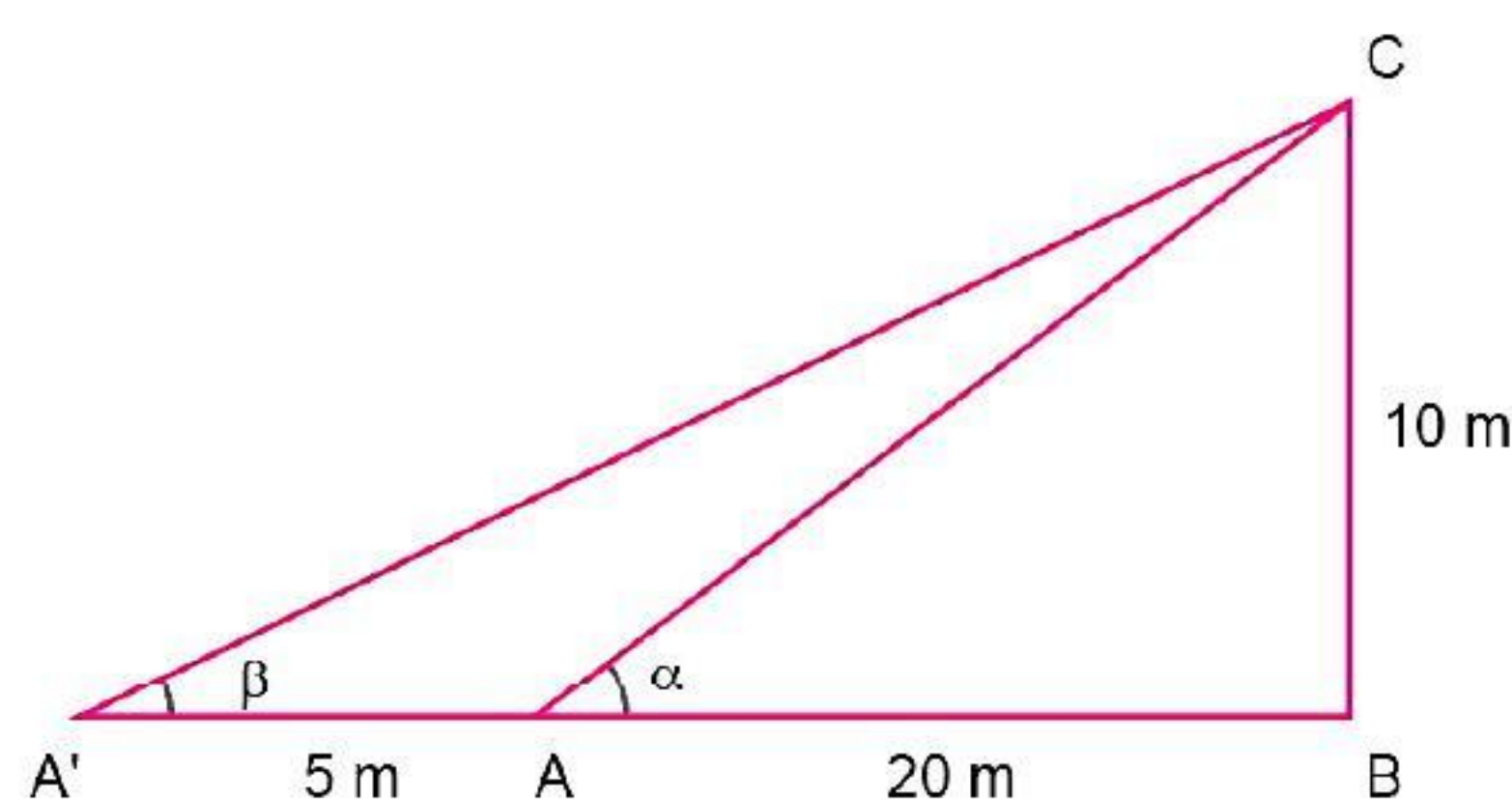
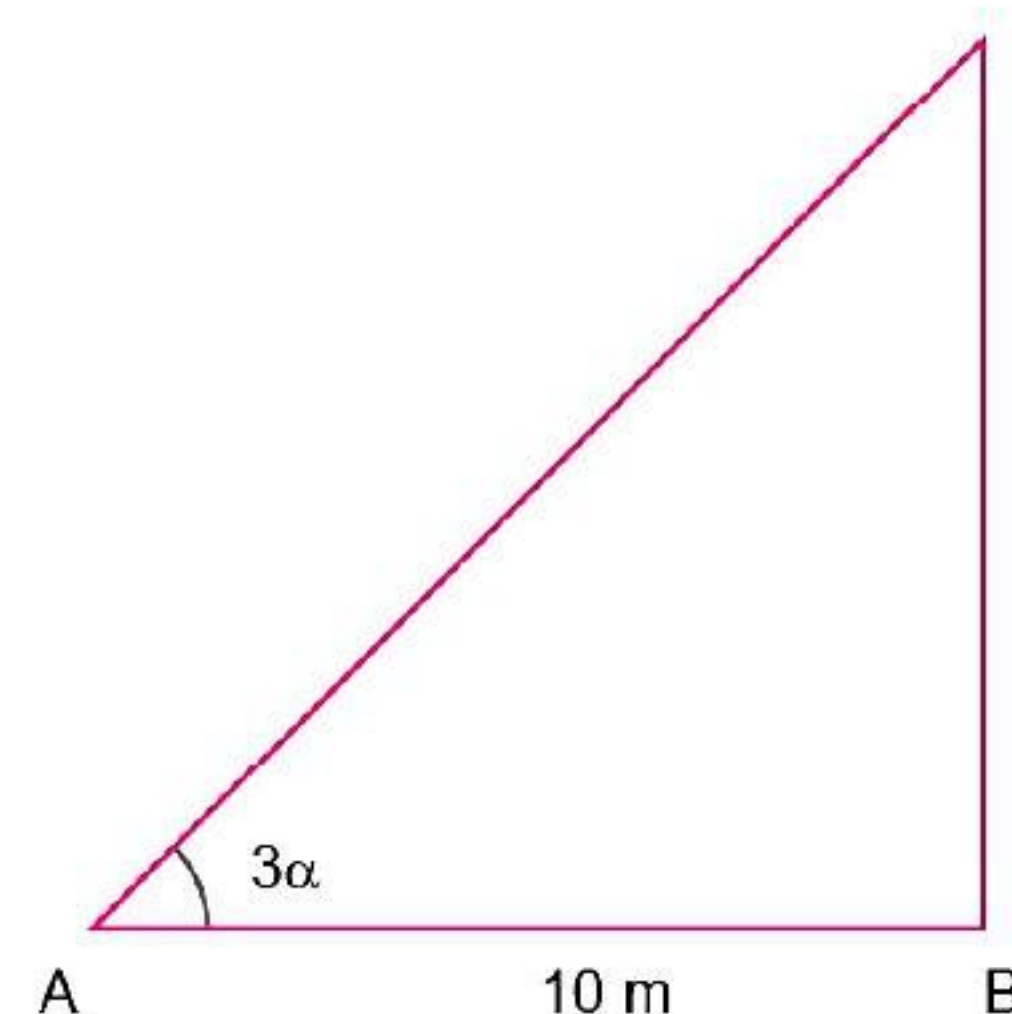
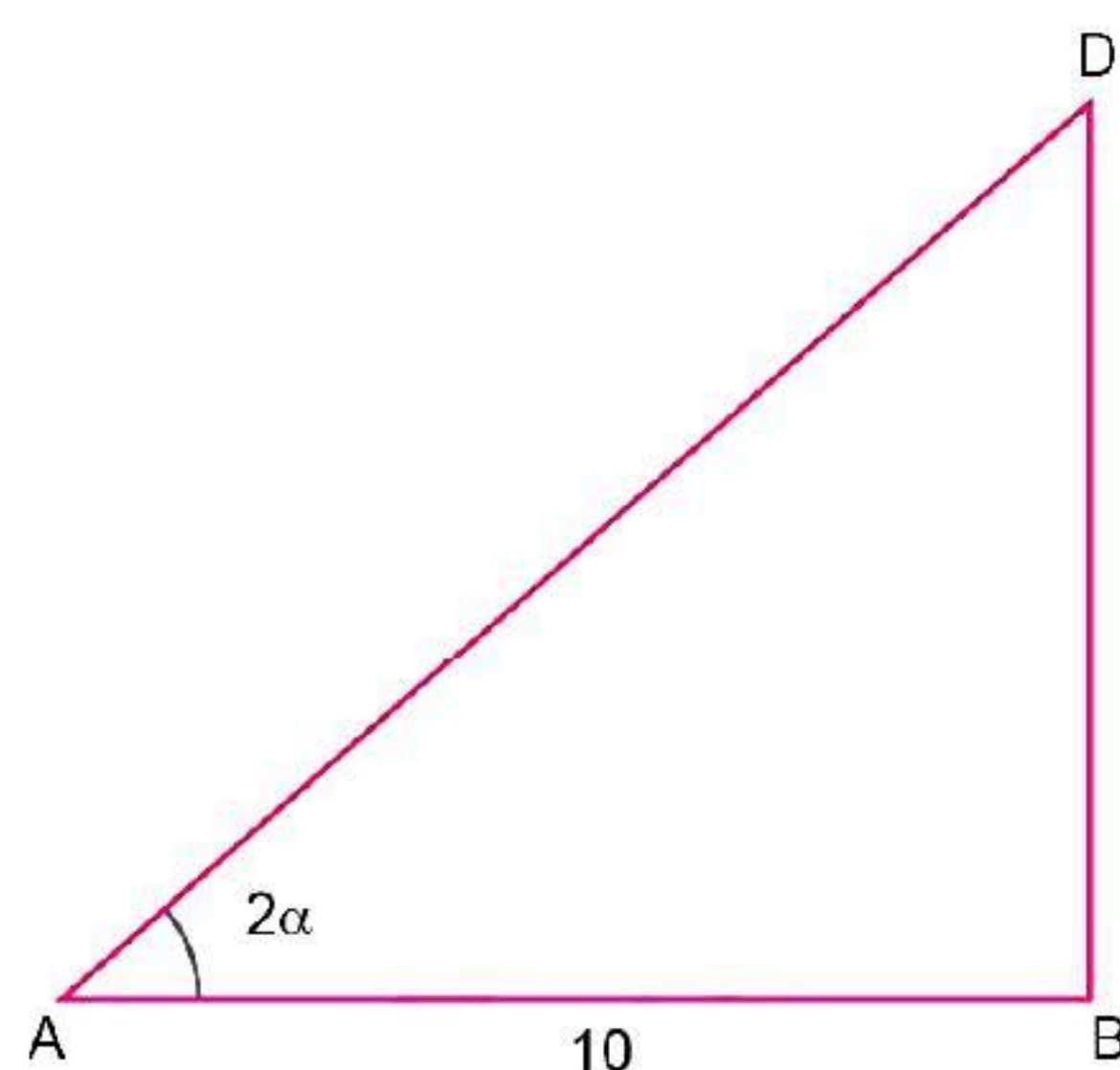
$$\Rightarrow \tan \beta = \frac{10}{25} = \frac{2}{5}$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{2}{5}\right)$$

$$\Rightarrow \angle CA'B = \tan^{-1}\left(\frac{2}{5}\right) \text{ and we already know } \angle CAB = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \angle CAB - \angle CA'B = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{2}{5}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{2} - \frac{2}{5}}{1 + \frac{1}{2} \times \frac{2}{5}}\right) = \tan^{-1}\left(\frac{\frac{5-4}{10}}{\frac{12}{10}}\right)$$



$$= \tan^{-1}\left(\frac{1}{12}\right)$$

∴ Option (a) is correct.

(v) Domain and Range of $\tan^{-1} x$ are respectively $(-\infty, \infty)$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ i.e. $\mathbb{R}, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

∴ Option (c) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false and R is also false.

1. **Assertion (A) :** Domain of $f(x) = \sin^{-1} x + \cos x$ is $[-1, 1]$.

Reason (R) : Domain of a function is the set of all possible values for which function will be defined.

2. **Assertion (A) :** Function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \sin x$ is not a bijection.

Reason (R) : A function $f: A \rightarrow B$ is said to be bijection if it is one-one and onto.

3. **Assertion (A) :** Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

Reason (R) : $\tan^{-1}: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}$, $\tan^{-1}(x)$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

4. **Assertion (A) :** $\sin^{-1}(-x) = -\sin^{-1} x$; $x \in [-1, 1]$

Reason (R) : $\sin^{-1}: [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is a bijection map.

Answers

1. (a) 2. (a) 3. (a) 4. (b)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have,

$$\begin{aligned} \tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right) &= \tan^{-1}\left(\tan \frac{\pi}{3}\right) + \cos^{-1}\left(-\cos \frac{\pi}{3}\right) \\ &= \frac{\pi}{3} + \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right] = \frac{\pi}{3} + \cos^{-1}\left(\cos \frac{2\pi}{3}\right) \\ &= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi \end{aligned}$$

Option (c) is correct.

2. Let $\sin^{-1} x = \theta$, then $\sin \theta = x$.

$$\Rightarrow \operatorname{cosec} \theta = \frac{1}{x} \quad \Rightarrow \operatorname{cosec}^2 \theta = \frac{1}{x^2}$$

$$\Rightarrow 1 + \cot^2 \theta = \frac{1}{x^2} \Rightarrow \cot \theta = \frac{\sqrt{1-x^2}}{x}$$

Option (d) is correct.

$$3. \sin^{-1}\left(\cos \frac{\pi}{9}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right)\right) = \sin^{-1}\left(\sin \frac{7\pi}{18}\right) = \frac{7\pi}{18}$$

Option (d) is correct.

$$\begin{aligned} 4. \quad \theta &= \sin^{-1}(\sin(-600)) = \sin^{-1}(-\sin 600) \\ &= -\sin^{-1}(\sin 600) = -\sin^{-1}(\sin(2 \times 360 - 120)) \\ &= -\sin^{-1}(-\sin 120) = \sin^{-1}(\sin 120^\circ) \\ &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \end{aligned}$$

Option (a) is correct.

5. We have,

$$\begin{aligned} 2 \sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right) &= 2 \sec^{-1}\left(\sec\left(\frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) \\ &= 2 \times \frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6} \end{aligned}$$

Option (b) is correct.

$$6. \tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$$

$$\text{Let } \sec^{-1} 2 = \alpha \Rightarrow \sec \alpha = 2 \operatorname{cosec}^{-1} 3 = \beta \Rightarrow \operatorname{cosec} \beta = 3$$

$$\begin{aligned} \therefore \tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3) &= \tan^2(\alpha) + \cos^2(\beta) \\ &= \sec^2 \alpha - 1 + \operatorname{cosec}^2 \beta - 1 = \sec^2 \alpha + \operatorname{cosec}^2 \beta - 2 \\ &= 4 + 9 - 2 = 11 \end{aligned}$$

Option (b) is correct.

$$7. \because f(x) = \sin^{-1} \sqrt{x-1}$$

$$\Rightarrow 0 \leq x-1 \leq 1 \Rightarrow 1 \leq x \leq 2$$

$$[\because \sqrt{x-1} \geq 0 \text{ and } -1 \leq \sqrt{x-1} \leq 1]$$

$$\therefore x \in [1, 2]$$

Option (a) is correct.

$$8. \cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right)$$

$$\text{Put } \cos^{-1}\left(\frac{7}{25}\right) = \alpha \Rightarrow \cos \alpha = \frac{7}{25}$$

$$\Rightarrow \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{625 - 49}{625}} = \frac{\sqrt{576}}{\sqrt{625}} = \frac{24}{25}$$

$$\therefore \cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \frac{7/25}{24/25} = \frac{7}{24}$$

Option (d) is correct.

$$9. \text{ Let } \tan^{-1} x = \alpha \Rightarrow \tan \alpha = x$$

$$\Rightarrow \sec \alpha = \sqrt{1 + \tan^2 \alpha} = \sqrt{1 + x^2}$$

$$\begin{aligned}\Rightarrow \cos \alpha &= \frac{1}{\sqrt{1+x^2}} \\ \therefore \sin \alpha &= \sqrt{1-\cos^2 \alpha} = \sqrt{1-\frac{1}{1+x^2}} = \sqrt{\frac{1+x^2-1}{1+x^2}} \\ &= \frac{x}{\sqrt{1+x^2}} \\ \therefore \sin(\tan^{-1} x) &= \sin(\alpha) = \frac{x}{\sqrt{1+x^2}}\end{aligned}$$

Option (d) is correct.

10. We have, $\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$

We know that, $0 \leq \cos^{-1} x \leq \pi$

$$\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$$

If and only if, $\cos^{-1} \alpha = \cos^{-1} \beta = \cos^{-1} \gamma = \pi$

$$\Rightarrow \cos \pi = \alpha = \beta = \gamma \Rightarrow -1 = \alpha = \beta = \gamma$$

$$\Rightarrow \alpha = \beta = \gamma = -1$$

$$\begin{aligned}\therefore \alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta) &= -1(-1 - 1) - 1(-1 - 1) - 1(-1 - 1) \\ &= 2 + 2 + 2 = 6\end{aligned}$$

Option (c) is correct.

11. $\tan^{-1} \sqrt{3} = \tan^{-1} \left(\tan \frac{\pi}{3} \right) = \frac{\pi}{3}$

$$\sec^{-1}(-2) = \sec^{-1} \left(-\sec \frac{\pi}{3} \right) = \sec^{-1} \left(\sec \left(\pi - \frac{\pi}{3} \right) \right)$$

$$= \sec^{-1} \left(\sec \frac{2\pi}{3} \right) = \frac{2\pi}{3}$$

$$\therefore \tan^{-1} \sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

Option (b) is correct.

12. Let $\cos^{-1} x = \theta$

$$\Rightarrow x = \cos \theta \Rightarrow \sec \theta = \frac{1}{x} \Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{x^2} - 1} \Rightarrow \tan \theta = \frac{1}{x} \sqrt{1 - x^2}$$

$$\text{Now, } \sin[\cot^{-1}\{\tan(\cos^{-1} x)\}] = \sin[\cot^{-1}\{\tan \theta\}]$$

$$= \sin \left[\cot^{-1} \left\{ \frac{1}{x} \sqrt{1 - x^2} \right\} \right]$$

$$\text{Again, let } x = \sin \alpha = \sin \left[\cot^{-1} \left\{ \frac{1}{\sin \alpha} \sqrt{1 - \sin^2 \alpha} \right\} \right]$$

$$= \sin \left[\cot^{-1} \left\{ \frac{\cos \alpha}{\sin \alpha} \right\} \right] = \sin[\cot^{-1}(\cot \alpha)]$$

$$= \sin \alpha = x$$

Option (c) is correct.

13. $\sin^{-1}(\sin 600)^\circ = \sin^{-1}[\sin(540^\circ + 60^\circ)]$

$$= \sin^{-1} \left[\sin \left(3\pi + \frac{\pi}{3} \right) \right] = \sin^{-1} \left[-\sin \frac{\pi}{3} \right]$$

$$= \sin^{-1} \left[\sin \left(-\frac{\pi}{3} \right) \right] = -\frac{\pi}{3}$$

Option (b) is correct.

$$\begin{aligned} 14. \quad \cos^{-1} \left(\cos \frac{7\pi}{6} \right) &= \cos^{-1} \left(\cos \left(2\pi - \frac{5\pi}{6} \right) \right) = \cos^{-1} \left(\cos \frac{5\pi}{6} \right) \\ &= \frac{5\pi}{6} \left[\because \frac{5\pi}{6} \in [0, \pi] \right] \end{aligned}$$

Option (b) is correct.

$$\begin{aligned} 15. \quad \sin \left[\frac{\pi}{3} - \sin^{-1} \left(-\frac{1}{2} \right) \right] &= \sin \left[\frac{\pi}{3} - \left(-\frac{\pi}{6} \right) \right] \left[\because \sin^{-1} \left(-\frac{1}{2} \right) = -\frac{\pi}{6} \right] \\ &= \sin \left(\frac{\pi}{3} + \frac{\pi}{6} \right) = \sin \frac{\pi}{2} = 1 \end{aligned}$$

Option (d) is correct.

$$\begin{aligned} 16. \quad &\because \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) \\ &= \tan^{-1}(\sqrt{3}) - (\pi - \cot^{-1}(\sqrt{3})) \\ &= \tan^{-1}(\sqrt{3}) - \pi + \cot^{-1}(\sqrt{3}) \\ &= \tan^{-1}(\sqrt{3}) + \cot^{-1}(\sqrt{3}) - \pi = \frac{\pi}{2} - \pi = -\frac{\pi}{2} \end{aligned}$$

Option (b) is correct.

$$17. \quad \text{Given, } y = \cos^{-1}(x^2 - 4)$$

$$\Rightarrow \cos y = x^2 - 4$$

$$\Rightarrow -1 \leq x^2 - 4 \leq 1 \quad \left[\because -1 \leq \cos y \leq 1 \right]$$

$$\Rightarrow -1 + 4 \leq x^2 - 4 + 4 \leq 1 + 4$$

$$\Rightarrow 3 \leq x^2 \leq 5$$

$$\Rightarrow x^2 \geq 3 \text{ and } x^2 \leq 5$$

$$\Rightarrow x^2 - 3 \geq 0 \text{ and } x^2 - 5 \leq 0$$

$$\text{Now, } x^2 - 3 \geq 0 \Rightarrow x^2 - (\sqrt{3})^2 \geq 0$$

$$\Rightarrow (x + \sqrt{3})(x - \sqrt{3}) \geq 0$$

$$\Rightarrow (x + \sqrt{3}) \geq 0, (x - \sqrt{3}) \geq 0 \text{ or } (x + \sqrt{3}) \leq 0, (x - \sqrt{3}) \leq 0$$

$$\Rightarrow x \geq -\sqrt{3}, x \geq \sqrt{3} \text{ or } x \leq -\sqrt{3}, x \leq \sqrt{3}$$

$$\Rightarrow x \geq \sqrt{3} \text{ or } x \leq -\sqrt{3}$$

$$\rightarrow x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty) \quad \dots (i)$$

$$\text{Again, } x^2 - 5 \leq 0 \Rightarrow x^2 - (\sqrt{5})^2 \leq 0$$

$$\Rightarrow (x + \sqrt{5})(x - \sqrt{5}) \leq 0$$

$$\Rightarrow (x + \sqrt{5}) \geq 0, (x - \sqrt{5}) \leq 0 \text{ or } (x + \sqrt{5}) \leq 0, (x - \sqrt{5}) \geq 0$$

$$\Rightarrow x \geq -\sqrt{5}, x \leq \sqrt{5} \text{ or } x \leq -\sqrt{5}, x \geq \sqrt{5}$$

$$\Rightarrow -\sqrt{5} \leq x \leq \sqrt{5} \text{ or } x \leq -\sqrt{5}, x \geq \sqrt{5} \text{ not possible}$$

$$\Rightarrow x \in [-\sqrt{5}, \sqrt{5}] \quad \dots (ii)$$

$$(i) \text{ and } (ii) \quad x \in \{(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)\} \cap [-\sqrt{5}, \sqrt{5}] \Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$$

Option (c) is correct.

18. The domain of $\cos$ function is $\mathbb{R}$ and the domain of $\sin^{-1}$ is $[-1, 1]$

$$\Rightarrow \text{The domain of } \sin^{-1} x + \cos x \text{ is } \mathbb{R} \cap [-1, 1] = [-1, 1]$$

Option (c) is correct.

19. Let $\sin^{-1}(0.8) = \theta \Rightarrow \sin \theta = 0.8$

$$\Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta} \Rightarrow \cos \theta = \sqrt{1 - (0.8)^2}$$

$$\Rightarrow \cos \theta = \sqrt{1 - 0.64} \Rightarrow \cos \theta = \sqrt{0.36}$$

$$\Rightarrow \cos \theta = 0.6$$

$$\therefore \sin(2 \sin^{-1}(0.8)) = \sin 2\theta = 2 \sin \theta \cos \theta = 2 \times 0.8 \times 0.6 = 0.96$$

Option (c) is correct.

20. $\therefore 1 \text{ rad} = 57.75^\circ \Rightarrow 5 \text{ rad} = 288.75^\circ$

$$\Rightarrow \frac{3\pi}{2} < 5 < \frac{5\pi}{2} \Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$$

$$\text{Now, } \therefore \sin^{-1}(\sin 5) > x^2 - 4x$$

$$\Rightarrow 5 - 2\pi > x^2 - 4x \Rightarrow x^2 - 4x + (2\pi - 5) < 0$$

$$\Rightarrow 2 - \sqrt{9 - 2\pi} < x < 2 + \sqrt{9 - 2\pi}$$

$$\left(\begin{array}{l} \therefore x^2 - 4x + (2\pi - 5) = 0 \\ \Rightarrow x = 2 \pm \sqrt{9 - 2\pi} \end{array} \right)$$

$$\Rightarrow x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$$

Option (d) is correct.

21. Since, $-1 \leq \sin^{-1} x \leq 1$ and given $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow x = y = z = 1$$

$$\text{Also, } \therefore f(1) = 1$$

$$\text{and } f(p+q) = f(p) \cdot f(q) \quad \forall p, q \in \mathbb{R}, \text{ then}$$

$$\Rightarrow f(2) = f(1+1) = f(1) \cdot f(1) = 1 \times 1 = 1$$

$$\Rightarrow f(3) = f(2+1) = f(2) \cdot f(1) = 1 \times 1 = 1$$

$$\begin{aligned} \text{Now, } x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} &= 1 + 1 + 1 - \frac{1+1+1}{1+1+1} \\ &= 3 - \frac{3}{1+1+1} = 3 - \frac{3}{3} = 3 - 1 = 2 \end{aligned}$$

Option (a) is correct.

22. Given $u = \cot^{-1}(\sqrt{\tan \alpha}) - \tan^{-1}(\sqrt{\tan \alpha})$

$$\text{Put } \sqrt{\tan \alpha} = x$$

$$\Rightarrow u = \cot^{-1}(x) - \tan^{-1}(x) = \frac{\pi}{2} - \tan^{-1} x - \tan^{-1}(x)$$

$$\begin{aligned}\Rightarrow u &= \frac{\pi}{2} - 2 \tan^{-1} x \Rightarrow 2 \tan^{-1} x = \frac{\pi}{2} - u \\ \Rightarrow \tan^{-1} x &= \frac{\pi}{4} - \frac{u}{2} \\ \Rightarrow x &= \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) \Rightarrow \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = x = \sqrt{\tan \alpha}\end{aligned}$$

Option (a) is correct.

23. $\cos^{-1} x > \sin^{-1} x \Rightarrow \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x$

$$\Rightarrow 2 \cos^{-1} x > \frac{\pi}{2} \Rightarrow \cos^{-1} x > \frac{\pi}{4}$$

$$\Rightarrow x < \cos\left(\frac{\pi}{4}\right) \left[\because \cos x \text{ is decreasing function in } \left[0, \frac{\pi}{2}\right] \right]$$

$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

Hence $x \in \left[-1, \frac{1}{\sqrt{2}}\right]$ [As domain of $\cos^{-1} x$ $[-1, 1]$]

Option (c) is correct.

24. $\because \sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$ is one-one and onto function.

$\therefore$ Domain of $f(x) = \sin^{-1} x$ is $[-1, 1]$.

Option (b) is correct.

25. Domain of $\sin^{-1} \theta$ is $[-1, 1]$ i.e., $\theta \in [-1, 1]$

$\therefore$ Domain of $\sin^{-1}(-x^2)$ is

$$-x^2 \in [-1, 1] \Rightarrow -1 \leq -x^2 \leq 1$$

$$\Rightarrow 1 \geq x^2 \geq 0 \Rightarrow x \in [-1, 1]$$

Domain of $\sin^{-1}(-x^2)$ is $[-1, 1]$.

Option (c) is correct.

26. We have

$$ax + b \sec(\tan^{-1} x) = c$$

$$\Rightarrow ax + b \sec(\sec^{-1}(\sqrt{1+x^2})) = c$$

$$\Rightarrow ax + b\sqrt{1+x^2} = c \Rightarrow b\sqrt{1+x^2} = c - ax$$

$$\Rightarrow b^2(1+x^2) = (c - ax)^2 = c^2 + a^2x^2 - 2acx$$

$$\Rightarrow (a^2 - b^2)x^2 - 2acx + (c^2 - b^2) = 0$$

$$\Rightarrow x = \frac{2ac \pm \sqrt{4a^2c^2 - 4(a^2 - b^2)(c^2 - b^2)}}{2(a^2 - b^2)}$$

Similarly,

$$ay + b \sec(\tan^{-1} y) = c$$

$$\Rightarrow y = \frac{2ac \pm \sqrt{4a^2c^2 - 4(a^2 - b^2)(c^2 - b^2)}}{2(a^2 - b^2)}$$

$$\therefore xy = \frac{c^2 - b^2}{a^2 - b^2} \text{ and } x + y = \frac{2ac}{a^2 - b^2}$$

$$\therefore \frac{x+y}{1-xy} = \frac{2ac}{a^2-c^2}$$

Option (c) is correct.

$$\begin{aligned} 27. \quad \alpha &= \tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right), \beta = \tan^{-1}\left(-\tan\left(\frac{2\pi}{3}\right)\right) \\ \Rightarrow \tan \alpha &= \tan \frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{4} \\ \beta &= \tan^{-1}\left(-\tan\left(\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right) \\ \Rightarrow \tan \beta &= \tan\left(-\frac{2\pi}{3}\right) = -\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{2\pi}{3}\right) \\ \Rightarrow \tan \beta &= \tan \frac{\pi}{3} \Rightarrow \beta = \frac{\pi}{3} \\ \therefore 4\alpha &= 3\beta \end{aligned}$$

So option (a) is correct option.

$$\begin{aligned} 28. \quad \text{If } x &= \frac{\sqrt{3}}{2}, \text{ then} \\ \tan\left(\frac{\sin^{-1}x + \cos^{-1}x}{2}\right) &= \tan\left(\frac{\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}}{2}\right) \\ &= \tan\left(\frac{\frac{\pi}{3} + \frac{\pi}{6}}{2}\right) = \tan\left(\frac{\frac{\pi}{2}}{2}\right) = \tan \frac{\pi}{4} = 1 \end{aligned}$$

Option (b) is correct.

$$\begin{aligned} 29. \quad \text{Let } f(x) &= \cos^{-1}[x] \\ \text{Now, domain of } g(x) &= \cos^{-1}x \text{ is the set} \\ \{x \mid -1 \leq x \leq 1\} &= [-1, 1] \\ \therefore \text{Domain of given function is } \{x \mid -1 \leq [x] \leq 1\} \\ [x] &= \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 0 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x < 2 \end{cases} \\ \therefore \text{Domain of } \cos^{-1}[x] &\text{ is } [-1, 2). \end{aligned}$$

Option (c) is correct.

$$\begin{aligned} 30. \quad \text{We have} \\ f(x) &= \sin 2x + \cos 2x, g(x) = x^2 - 1 \\ \therefore g(f(x)) &= (f(x))^2 - 1 = (\sin 2x + \cos 2x)^2 - 1 \\ &= \sin^2 2x + \cos^2 2x + 2 \sin 2x \times \cos 2x - 1 \\ &= 1 + \sin 4x - 1 = \sin 4x \\ \text{Now, } \sin 4x &\text{ will be invertible for } -\frac{\pi}{2} \leq 4x \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{8} \leq x \leq \frac{\pi}{8} \\ \text{i.e. Domain of } g(f(x)) &\text{ for which it is invertible is } \left[-\frac{\pi}{8}, \frac{\pi}{8}\right] \\ \text{Option (b) is correct.} \end{aligned}$$

$$\begin{aligned} 31. \quad \text{Domain of } \sin^{-1}x &\text{ is } [-1, 1]. \\ \therefore \text{Domain of } \sin^{-1}[x] &\text{ is } \{x \mid -1 \leq [x] \leq 1\} \end{aligned}$$

$$\text{But } [x] = \begin{cases} -1 & \forall -1 \leq x < 0 \\ 0 & \forall 0 \leq x < 1 \\ 1 & \forall 1 \leq x < 2 \end{cases}$$

$\therefore$ Domain of $\sin^{-1} [x]$ is $[-1, 2)$

Option (b) is correct.

32. We have $\sin(\pi \cos x) = \cos(\pi \sin x)$

$$\Rightarrow \sin(\pi \cos x) = \sin(\pi/2 \pm \pi \sin x)$$

$$\Rightarrow \pi \cos x = \frac{\pi}{2} \pm \pi \sin x \Rightarrow \pi \cos x \mp \pi \sin x = \frac{\pi}{2}$$

$$\Rightarrow \cos x \mp \sin x = \frac{1}{2}$$

Squaring both sides, we get

$$1 \pm \sin 2x = \frac{1}{4} \Rightarrow \pm \sin 2x = \frac{1}{4} - 1$$

$$\Rightarrow \pm \sin 2x = -\frac{3}{4} \Rightarrow \sin 2x = \pm \frac{3}{4}$$

$$\Rightarrow x = \frac{1}{2} \sin^{-1}\left(\pm \frac{3}{4}\right) \Rightarrow x = \pm \frac{1}{2} \sin^{-1}\left(\frac{3}{4}\right)$$

Option (a) is correct.

33. $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$

$$\Rightarrow \cos^{-1}\left(x \times \frac{y}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}}\right) = \alpha$$

$$\Rightarrow \frac{xy}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\Rightarrow \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha - \frac{xy}{2}$$

Squaring both the sides, we get

$$(1-x^2)\left(1-\frac{y^2}{4}\right) = \left(\cos \alpha - \frac{xy}{2}\right)^2$$

$$\Rightarrow 1 - \frac{y^2}{4} - x^2 + \frac{x^2 y^2}{4} = \cos^2 \alpha + \frac{x^2 y^2}{4} - xy \cos \alpha$$

$$\Rightarrow 1 - \frac{y^2}{4} - x^2 = \cos^2 \alpha - xy \cos \alpha \Rightarrow 4 - y^2 - 4x^2 = 4 \cos^2 \alpha - 4xy \cos \alpha$$

$$\Rightarrow 4 - 4 \cos^2 \alpha = 4x^2 + y^2 - 4xy \cos \alpha$$

$$\Rightarrow 4(1 - \cos^2 \alpha) = 4x^2 + y^2 - 4xy \cos \alpha$$

$$\Rightarrow 4x^2 - 4xy \cos \alpha + y^2 = 4 \sin^2 \alpha$$

Option (d) is correct.

34. $3 \sin x + 4 \cos x = y^2 - 2y + 6 = (y-1)^2 + 5$

Which is possible only when $y-1=0 \Rightarrow y=1$

($\because$ Maximum value of LHS = 5 and Minimum value of RHS = 5)

$$\therefore 3 \sin x + 4 \cos x = 5$$

Put $3 = r \cos \theta$, $4 = r \sin \theta$ such that $9 + 16 = r^2$

$$\Rightarrow r^2 = 25 \Rightarrow r = 5. \text{ Also } \tan \theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\Rightarrow r \cos \theta \sin x + r \sin \theta \cos x = 5$$

$$\Rightarrow r \sin(x + \theta) = r \Rightarrow \sin(x + \theta) = 1$$

$$\Rightarrow x + \theta = \sin^{-1}(1) = \frac{\pi}{2}$$

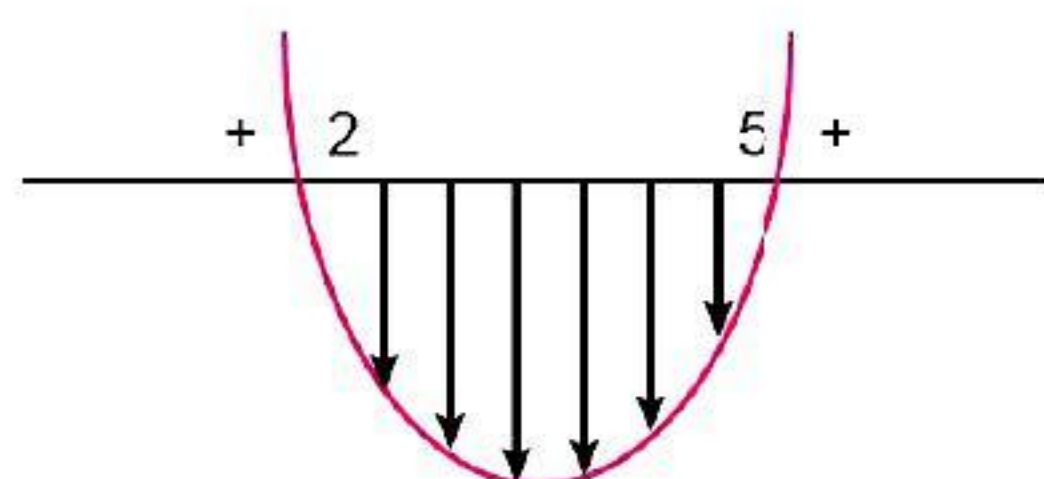
$$\Rightarrow x = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore xy = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$$

So all option are true.

Option (d) is correct.

35. $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$



$$\Rightarrow (\cot^{-1}x)^2 - 2 \cot^{-1}x - 5 \cot^{-1}x + 10 > 0$$

$$\Rightarrow (\cot^{-1}x)(\cot^{-1}x - 2) - 5(\cot^{-1}x - 2) > 0$$

$$\Rightarrow (\cot^{-1}x - 2)(\cot^{-1}x - 5) > 0$$

$$\Rightarrow \cot^{-1}x < 2, \cot^{-1}x > 5 \quad [\because \cot^{-1}x \text{ is decreasing } \forall x \in \mathbb{R}]$$

$$\Rightarrow x \in (-\infty, \cot 5) \cup (\cot 2, \infty)$$

Option (b) is correct.

36. Domain of $\sin^{-1}x$ is $[-1, 1]$ and domain of $\frac{1}{x}$ is $\mathbb{R} - \{0\}$

$$\therefore \text{Domain of } f(x) = \frac{\sin^{-1}x}{x} \text{ is } [-1, 1] - \{0\}$$

Option (d) is correct.

37. $f(x) = e^{\cos^{-1} \sin\left(x + \frac{\pi}{3}\right)}$

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1} \sin\left(\frac{8\pi}{9} + \frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1} \sin\left(\frac{11\pi}{9}\right)} = e^{\cos^{-1} \sin\left(\frac{22\pi}{18}\right)}$$

$$= e^{\cos^{-1} \sin\left(\frac{9\pi}{18} + \frac{13\pi}{18}\right)} = e^{\cos^{-1} \sin\left(\frac{\pi}{2} + \frac{13\pi}{18}\right)}$$

$$= e^{\cos^{-1} \cos\left(\frac{13\pi}{18}\right)} = e^{\frac{13\pi}{18}}$$

Option (b) is correct.

38. As $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so $-1 \leq \sin \theta \leq 1$ but we have $0 < \sin \theta < 1$

$$\log_{\sin \theta} (\cos^2 \theta - \sin^2 \theta) = 2$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = (\sin \theta)^2 \text{ and } 0 < \sin \theta < 1$$

$$\Rightarrow \cos^2 \theta = 2 \sin^2 \theta \text{ where } \sin \theta \in (0, 1)$$

$$\therefore \tan^2 \theta = \frac{1}{2} \Rightarrow \tan \theta = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$$

$$\text{Also } \cos^2 \theta - \sin^2 \theta = \sin^2 \theta, \sin \theta \in (0, 1)$$

$$\Rightarrow 1 - 2 \sin^2 \theta = \sin^2 \theta, \sin \theta \in (0, 1)$$

$$\Rightarrow 1 = 3 \sin^2 \theta \text{ where } \sin \theta \in (0, 1)$$

$$\Rightarrow \sin^2 \theta = \frac{1}{3}, \sin \theta \in (0, 1)$$

$$\Rightarrow \sin \theta = \pm \frac{1}{\sqrt{3}}, \sin \theta \in (0, 1)$$

But $\sin \theta = -\frac{1}{\sqrt{3}}$ is not possible, as base of log is +ve and not equal to 1.

$$\therefore \sin \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right).$$

Option (b) is correct.

39. Given $\sin^{-1} a + \sin^{-1} b + \sin^{-1} c = \frac{3\pi}{2}$

$$\Rightarrow \sin^{-1} a = \frac{\pi}{2} = \sin^{-1} b = \sin^{-1} c \Rightarrow a = b = c = 1$$

Again $f(x+y) = f(x)f(y), \forall x, y \in \mathbb{R}$ and $f(2) = 2$

$$\therefore f(4) = f(2+2) = f(2)f(2) = 2 \times 2 = 4$$

$$f(6) = f(2+4) = f(2)f(4) = 2 \times 4 = 8$$

$$\begin{aligned} \therefore a^{f(2)} + b^{f(4)} + c^{f(6)} - \frac{3(a^{f(2)} \cdot b^{f(4)} \cdot c^{f(6)})}{a^{f(2)} + b^{f(4)} + c^{f(6)}} \\ = a^2 + b^4 + c^8 - \frac{3(a^2 \times b^4 \times c^8)}{a^2 + b^4 + c^8} = 1^2 + 1^4 + 1^8 - \frac{3(1^2 \times 1^4 \times 1^8)}{1^2 + 1^4 + 1^8} \\ = 3 - \frac{3 \times 1}{3} = 3 - 1 = 2 \end{aligned}$$

Option (a) is correct.

40. We have

$$\sin^{-1}(x^2 - 7x + 12) = n\pi \Rightarrow x^2 - 7x + 12 = \sin(n\pi), n \in \mathbb{Z}$$

$$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x^2 - 3x - 4x + 12 = 0$$

$$\Rightarrow x(x-3) - 4(x-3) = 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow x = 3, 4$$

Option (b) is correct.

41. We have $[\tan^{-1} x]^2 - 2[\tan^{-1} x] - 3 \leq 0$

$$\text{Put } [\tan^{-1} x] = \alpha$$

$$\Rightarrow \alpha^2 - 2\alpha - 3 \leq 0$$

$$\Rightarrow \alpha^2 + \alpha - 3\alpha - 3 \leq 0 \Rightarrow \alpha(\alpha+1) - 3(\alpha+1) \leq 0$$

$$\Rightarrow (\alpha+1)(\alpha-3) \leq 0$$



$$\Rightarrow -1 \leq \alpha \leq 3 \Rightarrow -1 \leq [\tan^{-1} x] \leq 3$$

$$\Rightarrow -1 \leq \tan^{-1} x < 4$$

$$\Rightarrow \tan(-1) \leq x < \tan 4 < \infty \Rightarrow -\tan 1 \leq x < \infty$$

$$\Rightarrow x \in [-\tan 1, \infty)$$

Option (a) is correct.

$$\begin{aligned} 42. \quad \cos^{-1} \left[\cos \left(\left(-\frac{17}{15} \right) \pi \right) \right] &= \cos^{-1} \left[\cos \left(\frac{17\pi}{15} \right) \right] \\ &= \cos^{-1} \left[\cos \left(2\pi - \frac{13\pi}{15} \right) \right] = \cos^{-1} \left[\cos \frac{13\pi}{15} \right] = \frac{13\pi}{15} \end{aligned}$$

Option (b) is correct.

$$43. \quad \text{Principal value branch of } \cos^{-1} x \text{ is } [0, \pi]$$

Option (c) is correct.

$$44. \quad \text{Principal value branch of } \operatorname{cosec}^{-1} x \text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$$

Option (d) is correct.

$$\begin{aligned} 45. \quad \sin^{-1} \left[\cos \left(\frac{33\pi}{5} \right) \right] &= \sin^{-1} \left[\cos \left(6\pi + \frac{3\pi}{5} \right) \right] \\ &= \sin^{-1} \left[\cos \left(\frac{3\pi}{5} \right) \right] \\ &= \sin^{-1} \left[\sin \left(\frac{\pi}{2} - \frac{3\pi}{5} \right) \right] = \sin^{-1} \left[\sin \left(-\frac{\pi}{10} \right) \right] \\ &= \sin^{-1} \left[-\sin \left(\frac{\pi}{10} \right) \right] = -\sin^{-1} \left[\sin \left(\frac{\pi}{10} \right) \right] \\ &= -\frac{\pi}{10}, \because \frac{\pi}{10} \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \end{aligned}$$

Option (d) is correct.

$$\begin{aligned} 46. \quad \text{Let } \alpha &= \cos^{-1}(2x - 1) \Rightarrow \cos \alpha = 2x - 1 \\ \therefore -1 &\leq \cos \alpha \leq 1 \Rightarrow -1 \leq 2x - 1 \leq 1 \\ \Rightarrow 0 &\leq 2x \leq 2 \Rightarrow 0 \leq x \leq 1 \Rightarrow x \in [0, 1] \end{aligned}$$

Hence domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$

Option (a) is correct.

$$\begin{aligned} 47. \quad \cos^{-1} \left(\cos \frac{3\pi}{2} \right) &= \cos^{-1} \left(\cos \left(\pi + \frac{\pi}{2} \right) \right) \\ &= \cos^{-1} \left(-\cos \frac{\pi}{2} \right) = \cos^{-1}(0) = \frac{\pi}{2} \end{aligned}$$

Option (a) is correct.

$$48. \quad 2 \sec^{-1} 2 + \sin^{-1} \left(\frac{1}{2} \right) = 2 \times \frac{\pi}{3} + \frac{\pi}{6}$$

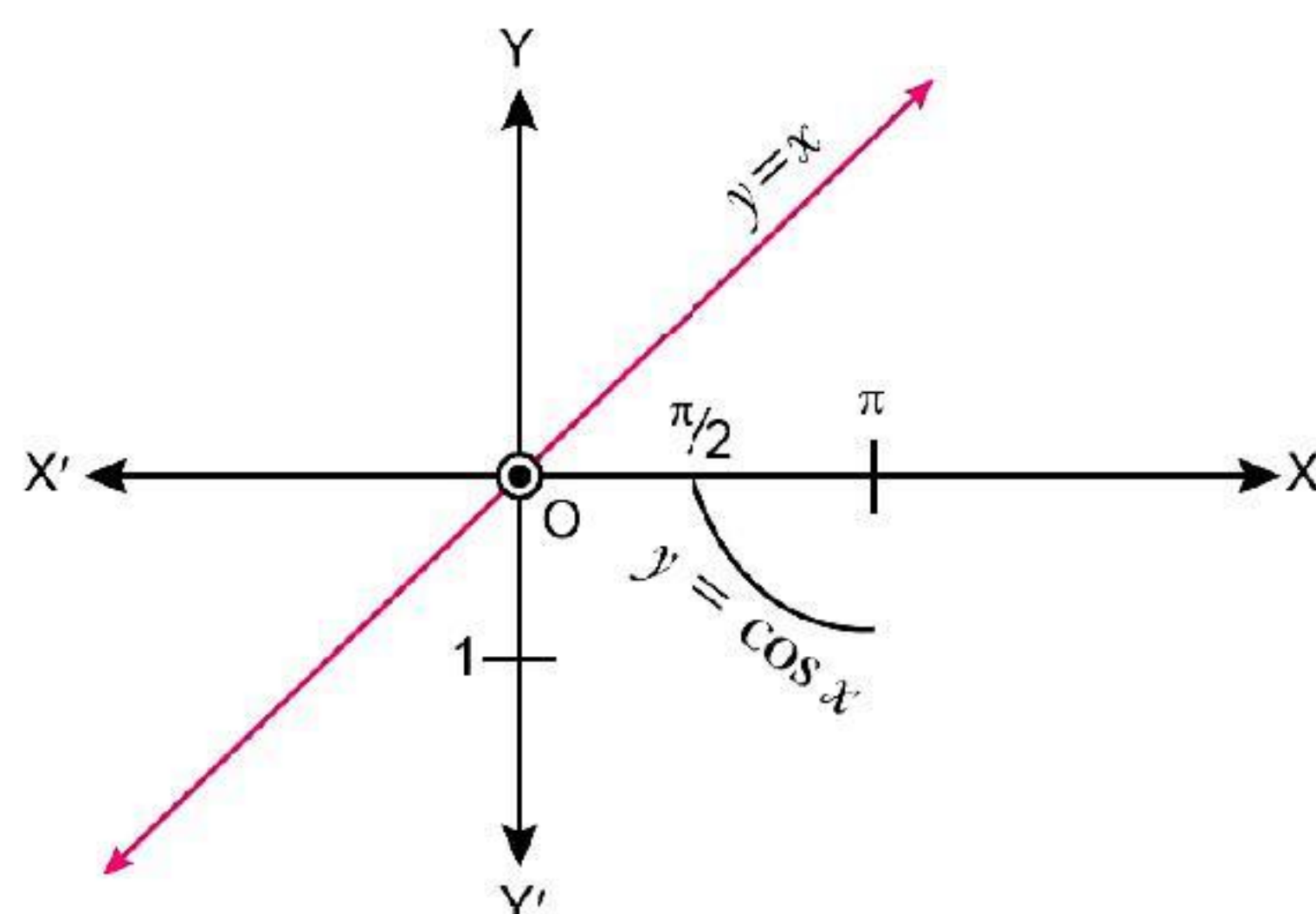
$$= \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$$

Option (b) is correct.

$$\begin{aligned} 49. \quad \cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right) &= \cot\left(\cot^{-1}\left(\frac{7/25}{\sqrt{1-(7/25)^2}}\right)\right) \\ &= \cot\left(\cot^{-1}\left(\frac{7/25}{24/25}\right)\right) = \cot\left(\cot^{-1}\left(\frac{7}{24}\right)\right) \\ &= \frac{7}{24} \end{aligned}$$

Option (d) is correct.

$$50. \quad \sqrt{1 + \cos 2x} = \sqrt{2} \cos^{-1}(\cos x)$$



$$\Rightarrow \sqrt{2} \cos^2 x = \sqrt{2} x, x \in \left[\frac{\pi}{2}, \pi\right] \subset [0, \pi]$$

$$\Rightarrow \sqrt{2} \cos x = \sqrt{2} x$$

$$\Rightarrow \cos x = x, \quad \because x \in \left[\frac{\pi}{2}, \pi\right]$$

As in the given figure graph of $y = x$ and $y = \cos x$ does not intersect in $\left[\frac{\pi}{2}, \pi\right]$ so the number of solutions is zero.

Option (a) is correct.

$$51. \quad \sin^{-1} \frac{1}{2} = \alpha, \text{ say } \Rightarrow \sin \alpha = \frac{1}{2} = \sin \frac{\pi}{6}$$

$$\Rightarrow \alpha = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow \text{Principal value of } \sin^{-1} \frac{1}{2} \text{ is } \frac{\pi}{6}$$

Option (a) is correct.

$$52. \quad \text{Let } \operatorname{cosec}^{-1}(-1) = \alpha \Rightarrow \operatorname{cosec} \alpha = -1 = \operatorname{cosec}\left(-\frac{\pi}{2}\right)$$

$$\Rightarrow \alpha = -\frac{\pi}{2} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$\therefore \text{Principal value of } \operatorname{cosec}^{-1}(-1) \text{ is } -\frac{\pi}{2}.$$

Option (a) is correct.

$$53. \quad \therefore \tan^{-1}(\sqrt{3}) + \cot^{-1}(-1) + \sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$$

$$\begin{aligned}
&= \frac{\pi}{3} + \pi - \cot^{-1}(1) + \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) \\
&= \frac{\pi}{3} + 2\pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{3} - \frac{5\pi}{12} \\
&= \frac{28\pi - 5\pi}{12} = \frac{23\pi}{12}
\end{aligned}$$

Option (d) is correct.

$$\begin{aligned}
54. \quad &2 \cos^{-1}\left(-\frac{1}{2}\right) + 2 \sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}(-1) \\
&= 2\left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) - 2 \sin^{-1}\left(\frac{1}{2}\right) - (\pi - \cos^{-1}1) \\
&= 2\left(\pi - \frac{\pi}{3}\right) - 2\left(\frac{\pi}{6}\right) - (\pi - 0) \\
&= 2\left(\frac{2\pi}{3}\right) - \frac{\pi}{3} - \pi \\
&= \frac{4\pi}{3} - \frac{\pi}{3} - \pi = \frac{4\pi - \pi - 3\pi}{3} = 0
\end{aligned}$$

Option (a) is correct.

$$\begin{aligned}
55. \quad &\sec^{-1}\left(\sec\frac{4\pi}{3}\right) = \sec^{-1}\left(\sec\left(\pi + \frac{\pi}{3}\right)\right) \\
&= \sec^{-1}\left(-\sec\frac{\pi}{3}\right) = \sec^{-1}(-2) = \pi - \sec^{-1}2 \\
&= \pi - \frac{\pi}{3} = \frac{2\pi}{3}
\end{aligned}$$

Option (b) is correct.

$$56. \quad \cos^{-1}(-1) + \sin^{-1}(1) = \pi - \cos^{-1}(1) + \sin^{-1}1 = \pi - 0 + \frac{\pi}{2} = \frac{3\pi}{2} \quad \text{Option (d) is correct.}$$

$$\begin{aligned}
57. \quad &\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right) \\
&= \cos^{-1}\left(\cos\left(\pi + \frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\pi + \frac{2\pi}{3}\right)\right) \\
&= \cos^{-1}\left(-\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{2\pi}{3}\right) = \cos^{-1}\left(-\left(-\frac{1}{2}\right)\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \\
&= \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} - \frac{\pi}{3} = 0
\end{aligned}$$

Option (a) is correct.

$$58. \quad \cot\left[\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \cot\left[\frac{1}{2} \times \frac{\pi}{3}\right] = \cot^{-1}\left(\frac{\pi}{6}\right) = \sqrt{3}$$

Option (c) is correct.

$$\begin{aligned}
59. \quad &\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \cot^{-1}(-\sqrt{3}) + \operatorname{cosec}^{-1}(\sqrt{2}) + \tan^{-1}(-1) + \sec^{-1}(\sqrt{2}) \\
&= -\sin^{-1}\left(\frac{1}{2}\right) + \pi - \cos^{-1}\left(\frac{1}{2}\right) + \pi - \cot^{-1}(\sqrt{3}) + \operatorname{cosec}^{-1}(\sqrt{2}) - \tan^{-1}(1) + \sec^{-1}(\sqrt{2})
\end{aligned}$$

$$= -\frac{\pi}{6} + \pi - \frac{\pi}{3} + \pi - \frac{\pi}{6} + \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi}{4}$$

$$= -\frac{2\pi}{3} + \frac{9\pi}{4} = \frac{19\pi}{12}$$

Option (b) is correct.

60. $\cos^{-1}(2x^2 - 1)$

Put $x = \cos \alpha \Rightarrow \alpha = \cos^{-1} x$

$$\therefore \cos^{-1}(2x^2 - 1) = \cos^{-1}(2 \cos^2 \alpha - 1) = \cos^{-1}(\cos 2\alpha)$$

$$\left[\begin{array}{l} \because 0 \leq x \leq 1 \\ \Rightarrow \cos\left(-\frac{\pi}{2}\right) \leq \cos \alpha \leq \cos 0 \\ \Rightarrow -\frac{\pi}{2} \leq \alpha \leq 0 \\ \Rightarrow -\pi \leq 2\alpha \leq 0 \\ = 2\alpha = 2 \cos^{-1} x \end{array} \right]$$

Option (a) is correct.

